



Symbolab Limits Cheat Sheet

Limit Properties:

If the limit of $f(x)$, and $g(x)$ exists, then the following apply:

- $\lim_{x \rightarrow a} x = a$
- $\lim_{x \rightarrow a} (f(x))^c = (\lim_{x \rightarrow a} f(x))^c$
- $\lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$
- $\lim_{x \rightarrow a} [c \cdot f(x)] = c \cdot \lim_{x \rightarrow a} [f(x)]$
- $\lim_{x \rightarrow a} [f(x) \cdot g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$
- $\lim_{x \rightarrow a} \left[\frac{f(x)}{g(x)} \right] = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$, where $\lim_{x \rightarrow a} g(x) \neq 0$

Limit to Infinity Properties:

For $\lim_{x \rightarrow c} f(x) = \infty$, $\lim_{x \rightarrow c} g(x) = L$, the following applies:

- $\lim_{x \rightarrow c} [f(x) \pm g(x)] = \infty$
- $\lim_{x \rightarrow c} [f(x) \cdot g(x)] = \infty, L > 0$
- $\lim_{x \rightarrow c} [f(x) \cdot g(x)] = -\infty, L < 0$
- $\lim_{x \rightarrow c} \frac{g(x)}{f(x)} = 0$
- $\lim_{x \rightarrow \infty} ax^n = \infty, 0 < a$
- $\lim_{x \rightarrow -\infty} ax^n = \infty, n$ is even, $a > 0$
- $\lim_{x \rightarrow -\infty} ax^n = -\infty, n$ is odd, $a > 0$
- $\lim_{x \rightarrow \infty} \frac{c}{x^a} = 0$

Common Limits:

- $\lim_{x \rightarrow \infty} \left(1 + \frac{k}{x}\right)^x = e^k$
- $\lim_{x \rightarrow \infty} \left(\frac{x}{x+k}\right)^x = e^{-k}$
- $\lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e$

Limit Rules:

- Limit of a Constant: $\lim_{x \rightarrow a} c = c$
- Basic Limit: $\lim_{x \rightarrow a} x = a$
- Squeeze Theorem: Let f, g and h be functions such that for all $x \in [a, b]$ (except possibly at the limit point c), $f(x) \leq h(x) \leq g(x)$. Also suppose that $\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} g(x) = L$, then for any $c, a \leq c \leq b$, $\lim_{x \rightarrow c} h(x) = L$
- L'Hopital's Rule: For $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$, if $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{0}{0}$ or $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\pm \infty}{\pm \infty}$ then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$
- Divergence Criterion: If there exists two sequences, $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ with: $x_n \neq c, y_n \neq c$ and $\lim_{n \rightarrow \infty} x_n = \lim_{n \rightarrow \infty} y_n = c$, $\lim_{n \rightarrow \infty} f(x_n) \neq \lim_{n \rightarrow \infty} f(y_n)$, then $\lim_{x \rightarrow c} f(x)$ does not exist
- Limit Chain Rule: If $\lim_{u \rightarrow b} f(u) = L$, and $\lim_{x \rightarrow a} g(x) = b$, and $f(x)$ is continuous at $x = b$, Then: $\lim_{x \rightarrow a} f(g(x)) = L$